S40RTS: a degree-40 shear-velocity model for the mantle from new Rayleigh wave dispersion, teleseismic traveltime and normal-mode splitting function measurements

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Accepted 2010 November 5. Received 2010 November 4; in original form 2010 May 7

SUMMARY
We have developed model S40RTS of shear-velocity variation in Earth's mantle using a new collection of Rayleigh wave phase velocity, teleseismic body-wave traveltime and normal-mode splitting function measurements. This data set is an order of magnitude larger than used for S20RTS and includes new data types. The data are related to shear-velocity perturbations from the (anisotropic) PREM model via kernel functions and ray paths that are computed using PREM. Contributions to phase delays and traveltimes from the heterogeneous crust are estimated using model CRUST2.0. We calculate crustal traveltimes from long-period synthetic waveforms rather than using ray theory. Shear-velocity perturbations are parametrized by spherical harmonics up to degree 40 and by 21 vertical spline functions for a total of 35 301 degrees of freedom. S40RTS is characterised by 8000 resolved unknowns. Since we compute the exact inverse, it is straightforward to determine models associated with fewer or more unknowns by adjusting the model damping. S40RTS shares many characteristics with S20RTS because it is based on the same data types and similar modelling procedures. However, S40RTS shows more clearly than S20RTS the abrupt change in the pattern of shear-velocity heterogeneity across the 660-km phase transition and it presents a more complex pattern of shear-velocity heterogeneity in the lower mantle. Utilities to visualise S40RTS and software to analyse the resolution of S40RTS (or models for different damping parameters) are made available.

Key words: Composition of the mantle; Body waves; Surface waves and free oscillations; Seismic tomography.

1 INTRODUCTION
Seismic tomography is one of the most important tools for elucidating the a-spherical (3-D) structure of Earth's mantle. Since the earliest seismic models of the mantle have been developed (e.g. Dziewonski et al. 1977; Woodhouse & Dziewonski 1984), they have been key input in the geophysical interpretation of the geoid, topography, hotspot volcanism and large-scale mantle dynamics (e.g. Hager et al. 1985; Hager & Richards 1989; Richards & Engebretson 1992; Ricard et al. 1993). See Becker & Boschi (2002) and Romanowicz (2003) for reviews.

Apart from well-sampled subduction zone regions (van der Hilst et al. 1991; Fukao et al. 1992), it has remained difficult to place global constraints on the seismic structure of the transition zone and lower mantle above D". Here, seismic heterogeneity is relatively weak and small in scale. Moreover, overtine surface waves and multiple shear-wave reflections, ideal seismic signals for studying the transition zone, are excited primarily by relatively strong ($M_w > 7$) earthquakes at intermediate depths ($H > 50$ km) and their waveforms are complex due to wave interactions with the free surface and lithosphere.

Fig. 1 shows six maps of the shear-velocity structure at a depth of 600 km according to models from six independent research groups: S40RTS (this study), S362ANI (Kustowski et al. 2008), SAW642AN (Panning & Romanowicz 2006), TX2008 (Simmons et al. 2009) and HMsl-S (Houser et al. 2008). Taken at face value, the maps display significant differences even if we consider the continental-scale (~4000 km) velocity variations. Note, for example, the variable strengths of high-velocity anomalies beneath the western Pacific subduction zones and the variable widths of low-velocity anomalies beneath oceans and continents. While the transition zone may separate vigorous convection in the upper mantle and sluggish convection in the lower mantle, it is difficult to infer the role of the 660-km phase and viscosity transition on large-scale material flow and plume ascent on the basis of Fig. 1. It is necessary to improve models of the transition zone and lower mantle by maximising...
Figure 1. Comparison of shear-velocity maps at a depth of 600 km for models (from top to bottom) S40RTS (this study), S362ANI (Kustowski et al. 2008), SAW642AN (Panning & Romanowicz 2006), TX2008 (Simmons et al. 2009) and HMSL-S (Houser et al. 2008). The velocity varies from $-2\%$ to $+2\%$ from the average value.

In this paper, we describe a new model of shear-velocity variations in the mantle. This model, S40RTS, is a successor to S20RTS described by Ritsema et al. (1999) (RHW99 from hereon) and Ritsema et al. (2004) (RHW04 from hereon). The effort in developing S40RTS has focused on maximising data coverage of the mantle. It is well-known that seismic wave sampling of the deep mantle is heterogeneous because most earthquakes occur in narrow belts along plate boundaries and seismometers are primarily installed on land (e.g. Wysession 1996). Until it becomes practical to extend instrumental coverage into the oceans (e.g. Simons et al. 2009), we must exploit the rich information in seismograms to overcome the limited earthquake and seismic station distribution. In principle, it is possible to relate any portion of the seismogram to seismic structure via adjoint methods (Liu & Tromp 2006, 2008). However, we follow here a classical approach in which nearly all high-amplitude signals in seismograms are identified as body-wave or surface-wave phases or normal-mode spectral peaks and analysed via waveform modeling.

The parametrization of the mantle and the modelling strategies used in S20RTS and S40RTS are very similar. Waveform anomalies (e.g. traveltime delays, phase delays and normal-mode splitting) are identified using Preliminary Reference Earth Model (PREM) (Dziewonski & Anderson 1981) synthetics and related to 3-D shear-velocity heterogeneity with ray paths and sensitivity kernels computed using PREM. Thus, S40RTS is a model of 3-D perturbations of isotropic shear velocity with respect to the PREM model. We invert the data using a damped least-squares inversion scheme. By computing the inverse of the GTG matrix, it is straightforward to investigate models of shear velocity for any damping factor and to determine the accompanying model resolution matrices.

We focus on presenting S40RTS and leave detailed image interpretations for forthcoming studies. In Section 2, we describe the data sets used in S40RTS in some detail. We discuss the modelling procedures in Section 3, including a brief discussion of finite-frequency crustal corrections, the forward modelling approach and model parametrization. We describe S40RTS in Section 4 and compare it to other recently published global shear-velocity models in Section 5.

2 DATA

Akin to RHW99 and RHW04 we use three data types: Rayleigh-wave phase delays, traveltimes of teleseismic body waves and the splitting functions of normal modes. These data provide complementary constraints on the shear-velocity structure of the mantle. The upper third of the mantle is primarily constrained by Rayleigh-wave dispersion and the lower two thirds of the mantle are constrained by teleseismic traveltimes. Whole-mantle constraints, especially at the longest wavelengths, are obtained from normal-mode splitting functions. We have updated the waveform collection with seismograms from 2003–2008, analysed new body-wave phases and measured the splitting functions using the most recent earthquakes with moment magnitudes of 7.4 and higher. The data set is roughly 10 times larger than the data set used by RHW99.

2.1 Rayleigh wave phase velocities

We have used the mode-branch-stripping technique of van Heijst & Woodhouse (1997) to measure the dispersion of the fundamental
and first four overtone Rayleigh waves. In contrast to full wave-form modelling techniques, this technique allows us to measure the dispersion of overlapping overtone branches as well as the independent data with relatively high weights. As demonstrated in RHW04, the overtone Rayleigh-wave data are important for imaging the transition zone because they are sensitive to velocity variations well below 400 km depth. Moreover, overtone surface waves sample the transition zone much better than teleseismic body waves especially beneath poorly instrumented regions like the oceans.

The data set has been extended to include measurements for all earthquakes from the Global CMT catalog up to 2008 (Table 1). It includes over 20 million phase-velocity measurements for minor-arc and major-arc paths and they are derived from both vertical and horizontal-component long-period seismograms. The misfit (which includes misfit reduction by CRUST2.0) is lower than 10 per cent for the fundamental-mode Rayleigh-waves. Misfit is relatively high for the overtone Rayleigh waves because shear-velocity variations below a depth of 200 km of the mantle are relatively weak and the effects of the crust are smaller.

We refer the readers to van Heijst & Woodhouse (1999) for details on the overtone measurements and incorporated reliability estimates, to van Heijst & Woodhouse (1999) for a discussion of overtone phase-velocity maps, to RHW04 for the inversion for shear-velocity structure in the mantle and to Trampert & van Heijst (2002) for a discussion of azimuthal anisotropy in the transition zone based on our overtone measurements.

### 2.2 Teleseismic traveltimes

We use the procedure of Ritsema & van Heijst (2002) to measure traveltime anomalies of distinct body-wave phases with respect to PREM. This procedure involves a cross-correlation of low-pass filtered seismogram and synthetics (computed for PREM and CMT source parameters) within 80-s wide time windows around major body-wave phases. Automated checks ensure that the window includes only one high-amplitude body-wave phase. The quality of the measurements is quantified from measurements of noise, waveform fit, wave amplitude discrepancies and the amplitude of body-wave coda.

The data set includes 500 000 traveltime measurements (Table 2). In addition to SKS and SKKS, which are recorded on radial component seismograms, we have updated the set of traveltimes for S (and $S_{\text{diff}}$), SS and SSS measured on transverse component seismograms.

We have expanded the collection of traveltimes of multiple core reflections (e.g. $S_S$, $S_S$, $SS_S$) and surface reflections (e.g. SS, SSS) by expanding the epicentral distance range. Multiple core reflections are well recorded up to an epicentral distance of 110°. They have up to eight propagation segments in the mantle and traverse the lower mantle more steeply than S, SS, and SSS.

The traveltimes of multiple surface reflections that have propagated along the major great-circle arc (e.g. $SS_M$, $SSS_M$, $SSSS_M$) between distances of about 245°–280° are new data. Although their ray geometries are similar to SS, SSS, and SSSS (Fig. 2a), major-arc surface reflections sample different regions since they propagate in the opposite direction. Figs 2(b) and (c) compare the upper mantle sampling of the minor-arc SS phase to the sampling of the major-arc $SS_{SM}$. Since most seismic stations are in the northern hemisphere, SS reflects off the surface primarily in the northern hemisphere. On the other hand, most of the $SS_{SM}$ surface reflection points are within the southern oceans where the $SS_{SM}$ sampling is relatively poor.

The multiple core reflections and major-arc surface reflections have relatively small amplitudes due to their long propagation paths. In practise, most traveltimes of these phases are obtained for deep earthquakes with moment magnitudes larger than about 7 when these phases are not obscured by the short-period Love-wave train. Our data set includes 28 200 $S_S$, $S_S$, $SS_S$, $SS_S$, $SSM$ and $SSM$ traveltimes. Combined, these traveltimes represent about 10 per cent of the total number of traveltimes. We note, however, that 57 000 traveltime measurements is a significant set when compared to traveltime collections from the 1990s.

For all phases we also measure the traveltime of accompanying depth phases (e.g. $S_S$, $SS$, $SS_S$) if earthquakes are deeper than 150 km. Misfit is lowest for body waves that have multiple segments through the heterogeneous upper mantle (e.g. SS and SSS).

### 2.3 Splitting functions

Normal modes provide important constraints on the longest wavelength structure in the mantle (Fig. 3). In RHW99 and RHW04 we used the splitting function data up to 3 mHz from Resovsky & Ritzwoller (1999). The new data set has been entirely reprocessed because a number of large events have occurred since the most recent compilation of normal-mode splitting functions. Our new data set contains modal spectra up to 3 mHz for all $M_W > 7.6$ events from the last 32 years (1976–2008) and all $M_W > 7.4$ earthquakes deeper than 100 km. In particular, we now have continental earthquakes (including the 2008 $M_W$ 7.9 Wenchuan China, 1999 $M_W$ 7.6 Turkey and 2005 $M_W$ 7.6 Pakistan events) in addition to subduction zone earthquakes. The new data set also includes the
2004 December 26 Sumatra event and its 2005 March 25 after-
shock. A total of 92 new events provide significant increase of data
coverage in previously under-sampled regions. Our data set in-
cludes splitting functions of 49 spheroidal modes of the funda-
mental branch and the first five overtones derived from over 84 000
spectral–spectral measurements (Table 3). We exclude several 5th
overtone modes with large sensitivity to \( P \) velocity. In addition, we
exclude inner-core sensitive modes, which are strongly split due to
inner-core anisotropy (Woodhouse et al. 1986).

The splitting functions are measured from normal-mode spectra
using the approach of Resovsky & Ritzwoller (1999). We allowed
for cross-coupling between pairs of modes which are close in fre-
quency. We only use the even-degree self-coupling splitting func-
tions in our S40RTS inversion. The inversions for splitting functions
were started from PREM; we were able to obtain good convergence
without starting from a 3-D mantle model. Error bounds were de-
termined using cross-validation.

As our data set is significantly larger than the older data sets,
we were able to measure splitting functions with a higher precision
than was possible before. For example, mode \( s S_{12} \), which is strongly
sensitive to shear velocity in the upper mantle, was measured using
2668 spectral peaks (a ten-fold increase compared to previous data
sets) and we measured its splitting function up to angular order 20. The splitting data set will be discussed further by Deuss et al.
(2010).

3 MODELLING PROCEDURE

We follow closely the modelling procedure of RHW04 but note here
several important modifications.

3.1 Isotropic \( S \) velocity inversion

S40RTS is a model of isotropic shear-velocity perturbations from
PREM. The shear-velocity structure has a first-order effect on trav-
eltimes, normal-mode splitting and surface-wave dispersion. Al-
though there is abundant evidence for the presence of seismic
anisotropy in the mantle, we ignore anisotropy in our model in
contrast to recent work by Ekström & Dziewonski (1998), De-
bayle et al. (2004), Panning & Romanowicz (2006), Kustowski
et al. (2008) (see also the reviews by Montagner 1998, 2007).
Only modest additional misfit reduction can be attained when vari-
able anisotropy is incorporated in global models (e.g. Panning &
Romanowicz 2006). Moreover, the resolution of anisotropy in the
mantle is further complicated by the uncertain effects of the crust
on surface-wave dispersion (e.g. Bozdağ & Trampert 2008; Feirrer-
a et al. 2010).

We have experimented with inversions in which \( P \)-velocity vari-
ations are included in the upper 300 km of the mantle. The addition
of \( P \)-velocity variations helps to reduce the misfit of the long-period
surface waves and explains qualitatively the ocean-continent pattern
of \( PP \) traveltimes seen by Ritsema & van Heijst (2002). However, it
has no effect on the resolution of shear velocity. A more complete
analysis of the long-wavelength \( P \) structure from splitting functions
and \( P \) wave traveltimes will be presented in a forthcoming paper.

We calculate the sensitivity of splitting functions and Rayleigh-
wave dispersion to velocity variations using PREM. We as-
sume a constant scaling between shear velocity and density
\((\delta \ln \rho / \delta \ln V_S = 0.5)\) and we assume that the scaling between shear
velocity and \( P \) velocity \((\delta \ln V_S / \delta \ln V_P)\) increases from 2 at the
surface to 3 at the core–mantle boundary. This scaling provides op-
timal fit to teleseismic \( P \) wave traveltimes and \( P \)-sensitive splitting
functions.

3.2 Ray-based propagation paths

Apart from describing the effects of diffraction of waves with finite
wavelengths, new theoretical approaches allow us to in-
voke volumetric rather than ray-based, sensitivity kernels for body
waves (Dahlen et al. 2000; Nolet & Dahlen 2000; Zhao et al.
2000) and surface waves (Li & Romanowicz 1995; Marquer-
ing et al. 1996; Yoshizawa & Kennett 2002). Indeed, several au-
thors have measured body-wave traveltimes at multiple frequen-
cies and use the frequency-dependence of the sensitivity kernels
to improve tomographic inversions (Montelli et al. 2004; Siglchoh
Figure 3. Splitting functions for modes (left) $\gamma S_{12}$ and (right) $\gamma S_8$ which are sensitive to shear-velocity structure in the upper mantle and lower mantle, respectively. Shear wave sensitivity kernels of these modes are shown to the upper-right.

Table 3. Splitting function measurements.

<table>
<thead>
<tr>
<th>Modes</th>
<th># spectra</th>
<th>Misfit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{03}$-$S_{21}$</td>
<td>30 544</td>
<td>0.13</td>
</tr>
<tr>
<td>$S_{12}$-$S_{110}$-$S_{14}$</td>
<td>11 731</td>
<td>0.16</td>
</tr>
<tr>
<td>$S_{21}$-$S_{13}$</td>
<td>19 404</td>
<td>0.08</td>
</tr>
<tr>
<td>$S_{56}$-$S_{9}$</td>
<td>6 812</td>
<td>0.17</td>
</tr>
<tr>
<td>$S_{11}$-$S_{85}$</td>
<td>9 700</td>
<td>0.16</td>
</tr>
<tr>
<td>$S_{31}$-$S_{6}$</td>
<td>5 849</td>
<td>0.23</td>
</tr>
</tbody>
</table>

et al. 2008; Zaroli et al. 2010). Obviously, finite-frequency kernels describe the sensitivity of waves better than kernels based on ray-theory, especially when they are calculated for 3-D models (Chen et al. 2007; Fichtner et al. 2009; Tape et al. 2009). However, we apply ray-theory and the great-circle approximation to describe the propagation of body waves and surface waves. As in RHW99 and RHW04 (see also Kustowski et al. 2008) we relate phase-velocity anomalies, traveltime anomalies and splitting functions to shear-velocity heterogeneity by assuming that the Rayleigh wave is affected by structure in the plane of propagation, that the traveltime is affected by velocity heterogeneity along the geometrical ray and that splitting functions can be mapped into velocity heterogeneity using depth-dependent kernel functions. Ray theory is easily implemented and it is our view that the geophysical interpretations of models based on ray-theory and finite-frequency theory are, in essence, the same.

Montelli et al. (2004) show that there is a difference in the resolved amplitude of velocity anomalies if finite-frequency theory is used in body-wave tomography. However, they also demonstrate that maps derived from traveltime inversion using ray theory and finite-frequency theory correlate nearly perfectly. In Fig. 4, we compare fundamental-mode Rayleigh phase-velocity maps at periods of 50 s and 100 s that are derived with ray theory and with finite-frequency theory following Spetzler et al. (2002). Rayleigh waves at these periods are strongly sensitive to the crust and lithosphere where velocity heterogeneity is strong. Although these maps are derived using different amounts of damping, all maps are characterised by 1350 effective unknowns. The minor differences between the maps demonstrate that virtually identical structure can be derived from the great-circle approximation and finite-frequency theory if the regularization is adjusted. This has previously been demonstrated by Trampert & Spetzler (2006). For additional discussion, see Boschi (2003), van der Hilst & de Hoop (2005), Montelli et al. (2006) and Trampert & Spetzler (2006).

3.3 Crustal corrections

The crust has a profound influence on traveltimes. We make a-priori crustal corrections by subtracting the contribution to the Rayleigh-wave phase delay due to the crust. We correct the Rayleigh-wave phase and splitting-function measurements for crustal thickness, surface topography and sea level height according to model CRUST2.0 (Bassin et al. 2000).

We used ray theory in RHW99 and RHW04 to determine crustal traveltime corrections. Here, we calculate the crustal corrections from normal-mode synthetics for the 1-D profiles of CRUST2.0 (Ritsma et al. 2009). These corrections better represent the non-linear effects of the thin crust on seismic waves since propagation times are only a fraction of the relatively long ($T > 15$ s) wave period. Fig. 5 compares the one-way traveltime of shear waves through the crust according to ray theory and measurements using synthetic waveforms. It is evident that ray-theoretical and finite-frequency crustal traveltimes may differ by more than a second. For profile A0 of CRUST2.0, which is the 1-D profile for the oceanic crust excluding ridge and continental shelf regions, the ray theoretical shear wave crustal correction is $-0.87$ s while it is $-0.55$ s for a $S$ wave with a period of 20 s. The difference of 0.3 s may not appear significant for a single shear-wave segment through the crust. However, the difference is more than a second for a $SSS$ wave with two surface reflections within the oceans. This is substantial compared to the average traveltime delays.

3.4 Parametrization

We use the same spline functions as in RHW04 to parametrize vertical variation of shear velocity. The separation of the spline functions increases with depth. Relatively dense spline distribution helps to accommodate strong vertical shear-velocity variations across the lithosphere–asthenosphere boundary and the phase transition in the
Figure 4. Phase velocity maps of fundamental mode Rayleigh waves at (top) 100 s and (bottom) 50 s periods. The left column shows the maps using finite-frequency kernels following Spetzler et al. (2002), the middle column shows the maps derived using the great-circle approximation, and the right column shows the difference. The maps have been parametrized using spherical harmonics up to degree 60.

Figure 5. Ray-theoretical (x-axis) and ‘finite-frequency’ (y-axis) one-way shear-wave traveltimes through the crust determined for the 360 crustal profiles from CRUST2.0 (Bassin et al. 2000). The finite-frequency traveltimes are determined by cross-correlating normal-mode synthetics for PREM and CRUST2.0 synthetics following Ritsema et al. (2009).

transition zone albeit at the expense of the vertical resolution at the base of the mantle. We parametrize horizontal variation of $\delta V_s$ using spherical harmonics up to order 40.

4 MODEL S40RTS

The linear relationships between shear-velocity perturbations ($m$) and the traveltime, phase velocity and splitting measurements ($d$) can be written in matrix form:

$$d = Gm.$$  \(1\)

We estimate $m$ by damped least-squares inversion (Tarantola 1987; Menke 1989) in which we minimise the misfit $(Gm - d)^T (Gm - d) / d^T d$ and the model norm $\epsilon m^T m$, weighted by the dampings factor $\epsilon$. The solution to (1) is

$$m^\dagger = G^\dagger d,$$  \(2\)

where $G^\dagger$ is the generalized inverse of $G$. If $U \Lambda U^T$ is the eigen decomposition of $G^T G$, the generalised inverse is

$$G^\dagger = U \Lambda^{-1} U^T G^T,$$  \(3\)

where $\Lambda^{-1} = (\Lambda + \epsilon I)^{-1}$. Combining (1) and (2) yields

$$m^\dagger = R m^\prime,$$  \(4\)

where $R = G^\dagger G$ is the model resolution operator. The effective number of unknowns $N$ of model $m^\dagger$ is given by the trace of $R$:

$$N = \text{trace}(R).$$  \(5\)

Since we compute the eigen decomposition of $G^T G$, it is straightforward to determine models $m^\dagger$ for any damping factor $\epsilon$. A model $m^\dagger$ that is damped strongly (i.e. $\epsilon$ is large) is characterised by relatively few effective unknowns $N$ and it has small shear-velocity variations especially at the shortest scales. Fig. 6 shows shear-velocity heterogeneity for values of $N$ of 5000, 8000 and 11 000. As $N$ decreases (e.g. the damping factor $\epsilon$ increases), shear-velocity variations decrease. For example, the peak-to-peak shear-velocity variation at 2800 km depth is 4.7 per cent when $N = 8000$ but it drops to 3.4 per cent for $N = 5000$. The effect of damping is also evident from the amplitude spectra as a function of depth of these models (Fig. 7). While each model is characterised by a predominantly ‘red’ spectrum in the uppermost and lowermost mantle, an attribute that has been observed since the first tomographic images have been constructed (e.g. Su & Dziewonski 1991; Dziewonski et al. 2010), the decrease in spectral amplitude is strongest for the highest harmonic degrees (i.e. shortest wavelengths). This demonstrates that the smallest wavelength structures are the poorest constrained components of S40RTS.
Figure 6. Maps of shear-velocity heterogeneity at, from top to bottom, 100 km, 600 km, 1500 km and 2800 km depth for, from left to right, a total number of unknowns equal to $N = 5000$, $N = 8000$ and $N = 11000$ (eq. 5). Model S40RTS has 8000 unknowns.

Typically, $N$ is estimated from misfit curves and by cross-validation (Hastie & Tibshirani 1990). Fig. 8 illustrates how model misfit varies as a function of $N$. Shown is the misfit of fundamental-mode and overtone Rayleigh waves at a period of 62 s (Fig. 8a), the traveltimes of S, SS, ScS and SKS (Fig. 8b), and the combined splitting functions (Fig. 8c). As expected, misfit decreases when $N$ increases. The misfit is lowest for the fundamental mode Rayleigh wave which propagates through the strongly heterogeneous crust and uppermost mantle with well-resolved long-wavelength velocity heterogeneity. Misfit curves for phase delays for Rayleigh waves at other periods and the traveltimes of other body-wave phases have similar behaviour. For each data type, the decline in misfit is relatively small when $N$ is larger than 8000.

Selecting $N$ is, to large extent, subjective since we do not fully understand the measurement errors, the systematic errors originating from unmodelled crustal effects and the effects of theoretical simplifications. Ideally, model uncertainties are evaluated by the analysis of 3-D synthetics (Komatitsch et al. 2002; Qin et al. 2009; Bozdag & Tramper 2010). On the basis of the misfit curves of Fig. 8 and inspection of maps and cross sections we adopt S40RTS as the model with $N = 8000$ effective unknowns but we emphasize that the misfit varies little for models with $N$ between 5000 and 11 000.

4.1 Model images

Since S20RTS and S40RTS are based on the same data types and modelling approaches, it is not surprising that they correlate extremely well. Many of the model characteristics of S20RTS that we have discussed previously are still present in S40RTS. Although differences between S20RTS and S40RTS are subtle, they may have important implications for model interpretations.

Fig. 9 shows images of the upper mantle beneath the Atlantic and surrounding regions. Low velocity anomalies beneath the Mid-Atlantic Ridge, the Red Sea and East African Rift are narrower in S40RTS since lateral resolution is higher. The low-velocity anomaly beneath Iceland extends much deeper than elsewhere along the Mid-Atlantic Ridge (Montagner & Ritsema 2001) but it does not extend below the 660-km discontinuity. In S40RTS, this anomaly is significantly stronger (> 3 per cent) than in S20RTS and may inhibit a pure thermal explanation.
Shear velocities are relatively high in the upper mantle beneath the central Atlantic and along the Atlantic coasts of South America and Africa. We have previously associated these high-velocity anomalies with the Central Atlantic Minimum, a thermal anomaly proposed by Bonatti (1996) and with small-scale convection due to contrasting thickness of the continental and oceanic lithosphere (King & Ritsema 2000; Ritsema et al. 2004). In addition, the velocity anomaly linked to the West African Craton is significantly weaker in S40RTS.

The change in the pattern of shear-velocity variation across the 660-km discontinuity is much stronger in S40RTS. This change is obvious in the spectra of Fig. 7 and the maps of Fig. 9 illustrate that this is not confined to the subduction zone regions. The profound change of shear-velocity heterogeneity across the 660 km depth was previously reported by Gu et al. (2001) and Ritsema et al. (2004). It indicates that the 660-km boundary marks a change in seismic velocity heterogeneity between the upper and lower mantle.

Whole-mantle cross-sections in Fig. 10(a) show large-scale structure in the deep mantle beneath Africa and Asia. The African anomaly extends through the lower mantle in S20RTS and S40RTS but it is less obvious from S40RTS that this anomaly connects with the East African Rift anomaly in the upper mantle as Ritsema et al. (1999) speculated. The relatively 'sharp' top of the African anomaly is consistent with rapid traveltime variations (Ritsema et al. 1998) and wave multi-pathing (Ni et al. 2002) as seen in regional network data. Fig. 10(b) shows a cross-section through the Western Pacific through S20RTS and S40RTS. Here, S40RTS present clear indication of slab accumulation above the 660-km discontinuity and a more pronounced contrast of shear velocity heterogeneity across the 660-km discontinuity. A continuation of the subducting slab beneath Japan into the lower mantle is less obvious in S40RTS even though velocity heterogeneity in the lower mantle is generally better resolved and stronger than in S20RTS.
5 COMPARISON WITH OTHER MODELS

Fig. 11 (see also Fig. 1) compares S40RTS to other recently published shear-velocity models that are based on different data sets and modeling approaches: S362ANI (Kustowski et al. 2008), SAW642AN (Panning & Romanowicz 2006), TX2008 (Simmons et al. 2009) and HMSL-S (Houser et al. 2008). Model S362ANI is a whole-mantle shear-velocity model derived from surface-wave dispersion, mantle and body-wave waveforms and body-wave traveltimes. The data are inverted for isotropic (Voigt average) shear velocity and variations in shear-velocity anisotropy in the upper mantle. The authors also invert for a new 1-D reference model for shear and compressional velocity and anisotropy in the upper mantle. Model SAW642AN is a whole-mantle anisotropic model of the mantle derived from three-component surface-wave and body-wave waveforms. In contrast to all other models that are based on ray methods, SAW642AN is based finite-frequency kernels computed using the non-linear asymptotic coupling theory of Li & Romanowicz (1995). The data are inverted for isotropic (Voigt average) shear velocity and the parameter $\eta$ that represents the ratio of horizontally and vertically polarized shear wave speed. Model TX2008 is the seismic component of a joint seismic/geodynamic inversion. The shear-velocity structure is derived from 46 000
Figure 10. Whole-mantle cross-sections centered on (A) eastern Africa and (B) the western Pacific for models (top) S40RTS and (bottom) S20RTS. Shear-velocity perturbations are between −1.5 per cent and +1.5 per cent.

shear-wave traveltimes but surface wave data have not been included. Model HMSL-S is derived from S, SS, SS-S, ScS-S traveltimes and Love and Rayleigh waves.

The models correlate best in the uppermost mantle. Here, shear-velocity anomalies are relatively large (>15 per cent) and broad (>5000 km) and, hence, impart large delays in fundamental-mode surface waves and multiple S wave reflections. Although less clear for HMSL-S, shear-velocity heterogeneity is associated with thermal anomalies due to plate tectonics and the presence of stable continents. The lowest shear velocities are resolved beneath the fast-spreading East Pacific Rise and velocities increase with increasing plate age. Highest shear velocities are resolved beneath the oldest regions of continents.

The core–mantle boundary region is also a region with relatively strong (>4 per cent), long-wavelength variations in shear velocity. This region is constrained by normal-mode splitting and the traveltimes of diffracted S waves, ScS and SKS. Absolute and differential (e.g. ScS-S and S-SKS) traveltime delays can exceed 15 s, especially for paths crossing the lower mantle beneath the central Pacific and Africa. Since all of the models shown in Fig. 11 are based on traveltime sets that include S, ScS and SKS it is not surprising that the core–mantle boundary maps are similar.

Each of the models, apart from HMSL-S, indicates that there is a significant contrast between shear-velocity structure at 600 km and 800 km depth. The high-velocity structure beneath the western Pacific region, associated with subduction, is one of the most prominent structures just above the 660-km phase transition but not beneath the 660-km phase transition.

The differences between the models of Fig. 11 are in general larger than the differences between S20RTS and S40RTS. For example, models S40RTS and TX2008 present a contiguous low shear-velocity structure at 400 km depth beneath the Pacific Ocean at a depth of 400 km. This structure has a ring shape in models S362ANI and SAW642AN. Models S40RTS, SAW642AN, S362ANI and HMSL-S feature relatively high transition-zone velocities beneath the central Atlantic but these anomalies are missing in TX2008. The shear-velocity structure of SAW642AN at 400 km depth is anti-correlated from the structure at 100 km depth.

The model differences stem from differences in data sets and modelling approaches. Since teleseismic waves yield relatively poor sampling of the transition zone, its seismic resolution depends strongly on the modelling approaches (i.e. regularisation), on data coverage and whether overtone surface waves are analyzed as complete waveforms (e.g. S362ANI), as dispersion curves (e.g. S40RTS), or as triplicating, multiple S wave traveltimes (e.g. TX2008).

6 MODEL RESOLUTION

To explain model differences quantitatively, it is necessary to consider resolution tests that expose model inaccuracies. Here we show
Figure 11. Comparison of shear-velocity maps at depths of (from top to bottom) 100 km, 400 km, 600 km, 800 km, 1000 km, 2000 km and 2800 km for model (from left to right) S40RTS, S362ANI (Kustowski et al. 2008), SAW642AN (Panning & Romanowicz 2006), TX2008 (Simmons et al. 2009) and HMSL-S (Houser et al. 2008). The velocity varies from $-2$ per cent to $+2$ per cent from the average value. Shear-velocity perturbations are between $-7$ per cent and $+7$ per cent from the average value for the map at 100 km depth.

an example of the ‘checkerboard test’ in which a hypothetical structure is projected into the model space of S40RTS and filtered using eq. (4): $m^{OUT} = Rm^{IN}$. As test structures one often invokes checkerboard patterns of alternating high and low velocity anomalies or isolated spikes. In Fig. 12, we consider a geophysical test structure $m^{IN}$ determined by scaling the temperature variations in the mantle according to the numerical circulation model of Schuberth et al. (2009) into shear-velocity variations. Fig. 12(a) shows the shear-velocity variation for this model at a depth of a 1000 km. Figs 12(b), (c) and (d) shows the shear-velocity heterogeneity after filtering (i.e. $m^{OUT}$) for $N = 5000$, 8000 and 11000. Most of the ‘smearing’ and de-amplification of the original structure is a result of the parametrization of S40RTS. Relatively narrow slabs are expanded into spherical harmonics up to degree 40 which can only accommodate structures with a wavelength of 1000 km wavelength or more. Additional amplitude loss of the velocity variations is due the norm damping which increases with decreasing $N$.

A single test cannot provide a complete picture of the variable model resolution as discussed by Lévéque et al. (1993). However, the computation of Backus-Gilbert resolution kernels (e.g. Backus & Gilbert 1968) (see Ritsema et al. 2004, for examples) and tomographic filtering hypothesis tests for a wide variety of test structures can be readily performed using $R$ for S40RTS. Insights into the heterogeneous resolution of S40RTS and artefacts in S40RTS images can therefore be gained from a large number of checkerboard tests without relying on a limited number of published examples. Several examples have been discussed by Ritsema et al. (2007), Bull et al. (2009), Downey & Gurnis (2009) and Schuberth et al. (2009).

The resolution matrix does not account for systematic errors in S40RTS due to, for example, theoretical simplifications, the
effects of velocity anisotropy, or erroneous crustal corrections. However, $R$ exposes inaccuracies of S40RTS due to model parametrization, damping and incomplete data coverage which are important factors limiting image resolution.

7 DISCUSSION AND CONCLUSIONS

Using a new compilation of Rayleigh-wave phase velocity, body-wave traveltime and normal-mode splitting function measurements, we have constructed a new model of the 3-D shear-velocity structure of the mantle. Although we employ similar data types than used for S20RTS, S40RTS is derived from an order-of-magnitude larger data set and it offers higher spatial resolution.

Since we compute the decomposition of $G^T G$, it is straightforward to compute models with different dampings factors and to quantify model resolution. We select S40RTS to be the model that has 8000 effective unknowns, as defined by the trace of the model resolution matrix, on the basis of data misfit and the visual inspection of maps and cross-sections. This choice is subjective since it is not straightforward to estimate measurement errors and to determine the effects of erroneous crustal corrections and theoretical simplifications. However, following equation (3) is it straightforward to estimate models for different values of $\epsilon$ (and hence a different number of unknowns $N$) and to determine the model resolution operator $R$ as a function of damping $\epsilon$.

Model interpretation is often focused on the continuity of slabs and the presence of plumes in the mantle. However, the tomographic models from Fig. 11 disagree even at scales much larger than subducting slabs and rising plumes, demonstrating that the seismic structure of the transition zone and lower mantle is still inconsistently imaged. Model differences stem from choices in data processing and modelling approaches. Software to make maps, cross-sections and surface rendering plots are useful when comparing competing seismic models. It is also key that published models are accompanied by utilities that enable quantitative estimates of resolution in order to develop meaningful geophysical interpretations.

ACKNOWLEDGMENTS

JR has been funded by NSF grant EAR–1015172. AD was funded by the European Research Council (ERC) under the European Community’s Seventh Framework Programme (FP7/2007-2013), ERC grant agreement number 204995. Data have been provided by the IRIS/DMC and the Geoscope Data Center. Figures have been produced using the GMT software of Wessel & Smith (1995). We thank Christine Houser, Jeannot Trampert and an anonymous reviewer for constructive reviews.

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